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## Relationship Between Log Permeability and Fraction of Finer Grains in Bimodal Sediment Mixtures

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Relationship Between Log Permeability and Fraction of Finer Grains in  
Bimodal Sediment Mixtures

A thesis submitted in partial fulfillment  
of the requirements for the degree of  
Master of Science

By

STEVEN MARC VERDIBELLO  
B.S., The Ohio State University, 2010

2012  
Wright State University

WRIGHT STATE UNIVERSITY

GRADUATE SCHOOL

June 8, 2012

I HEREBY RECOMMEND THAT THE THESIS PREPARED UNDER MY  
SUPERVISION BY Steven M. Verdibello ENTITLED Relationship between log permeability  
and fraction of finer grains in bimodal sediment mixtures BE ACCEPTED IN PARTIAL  
FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF Master of Science.

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## ABSTRACT

Verdibello, Steven M., M.S., Department of Earth and Environmental Sciences, Wright State University, 2012. Relationship Between Log Permeability and Fraction of Finer Grains in Bimodal Sediment Mixtures.

In sediment mixtures of two grain-size components, the mixture porosity ( $\Phi$ ) and permeability ( $k$ ) both vary non-linearly as a function of the grain size and the volume fraction of each component. A porosity minimum ( $\Phi_{min}$ ) occurs near the mixture fraction at which the volume of the finer grains equals the original pore volume of the coarser grains. An abrupt change in slope has been observed in the non-linear relationship between  $\log(k)$  and the volume fraction of finer grains ( $r_f$ ). This slope change should occur at the  $r_f$  where coarser pore pathways change from continuous to discontinuous. In this study, fine sand was mixed with coarse sand at different volume fractions, and the abrupt slope change was observed to occur at an  $r_f$  near, but slightly above, the  $r_f$  at which  $\Phi_{min}$  occurred. Among published experiments that used a variety of grain sizes, the change in slope was observed to occur at  $r_f$  less than, equal to, or greater than the  $r_f$  at which  $\Phi_{min}$  occurs, but commonly occurs at  $r_f$  relatively close to that of  $\Phi_{min}$ .

For mixtures in which finer grains are approximately the size of coarser pores, unoccupied coarser pores will percolate (connect across the sample) if the fraction of occupied pores ( $\omega$ ) is below a percolation threshold ( $\omega_c$ ), and not percolate if  $\omega$  is greater than  $\omega_c$ . An abrupt change in slope of the  $\log(k)$  versus  $r_f$  relationship should occur at the  $r_f$  at which  $\omega$  equals  $\omega_c$ , which is less than the  $r_f$  at which the  $\Phi_{min}$  occurs. The concepts from percolation theory apply to mixtures in which finer grains are equal to or larger than the size of coarser pores, and do not apply to mixtures in which finer grains are much smaller than the coarser pores.

The literature contains three different methods for using the Kozeny-Carman equation to model the  $\log(k)$  versus  $r_f$  relationship based on ideas about the nature of the abrupt change in slope, and the  $r_f$  at which the change occurs. The Kozeny-Carman equation is robust and represents the relationship well using any of these methods.

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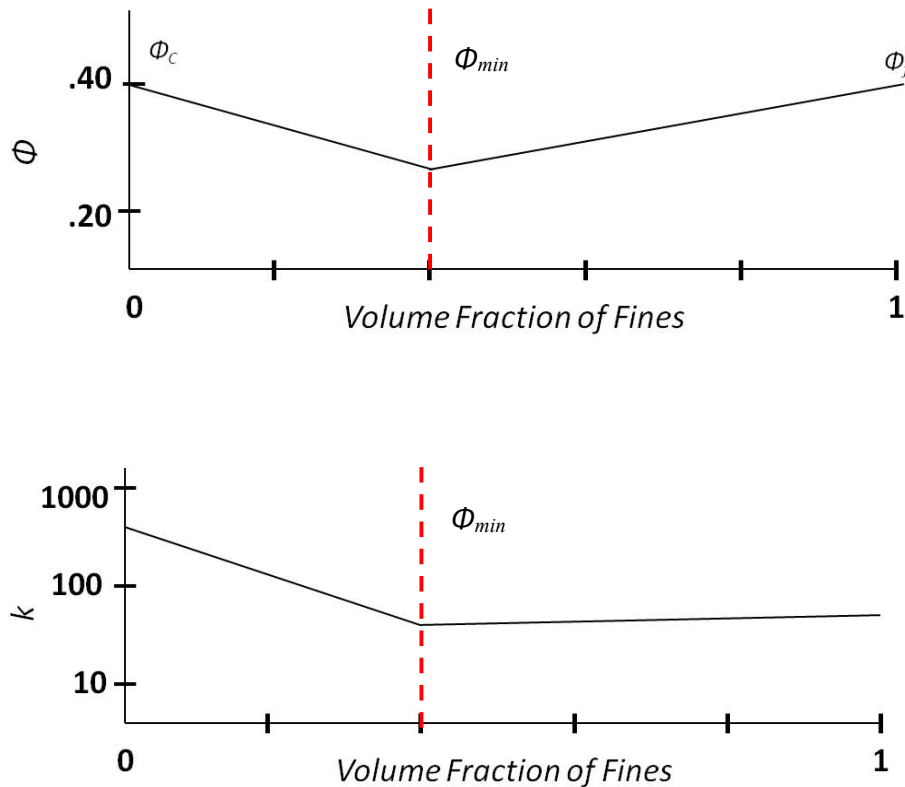
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## 1.0 Introduction

Permeability is an important hydraulic property governing subsurface fluid flow. Sediment grain size data are often more abundant than permeability data when characterizing environmental study sites or oil and gas reservoirs. Therefore, there is value in understanding how permeability is related to the distribution of grain sizes in sediment.

In mixtures of sediment comprising different grain sizes, porosity ( $\Phi$ ) and permeability ( $k$ ) vary non-linearly as a function of the volume fraction of finer-grained sediment mixed with coarser sediments (e.g. Shakoor and Cook, 1990; Esselburn et al., 2011; Porter et al., 2012). Figure 1 shows an idealized representation of these non-linear relationships for mixtures composed of two grain sizes.



**Figure 1.** A conceptual model of how porosity ( $\Phi$ ) and permeability ( $k$ ) vary with the proportion of finer grains in sediment mixtures composed of two grain sizes.

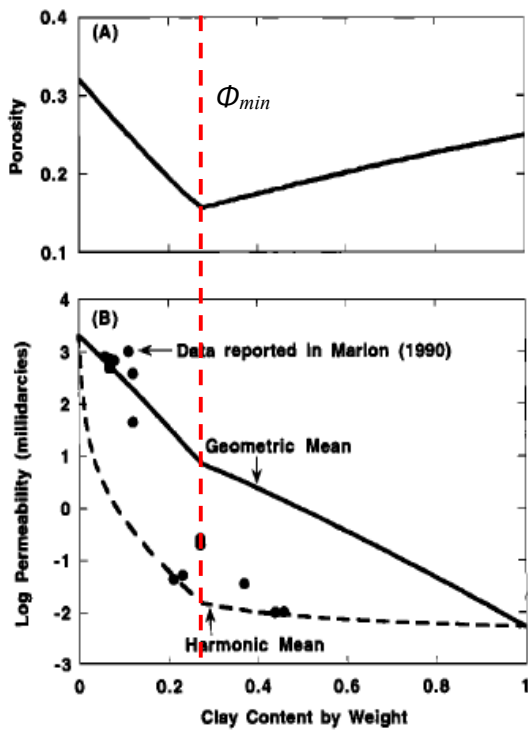


In an ideal mixture, the packing of sediment is uniform and can be defined as either coarse or fine (Clarke, 1979). Coarse packing occurs in mixtures where the coarse grains maintain their original packing and pore structure, and the finer grains fit into the coarse pore space. Conversely, fine packing occurs in mixtures where the fine grains maintain their original packing and act as a matrix, with the coarser grains interspersed in that matrix. The type of packing depends on the proportion of fine grains to coarse grains. Coarse packing exists if the volume of finer sediment is less than the volume of coarser-sediment pores, and fine packing exists if the volume of finer sediment is greater than the volume of coarser-sediment pores.

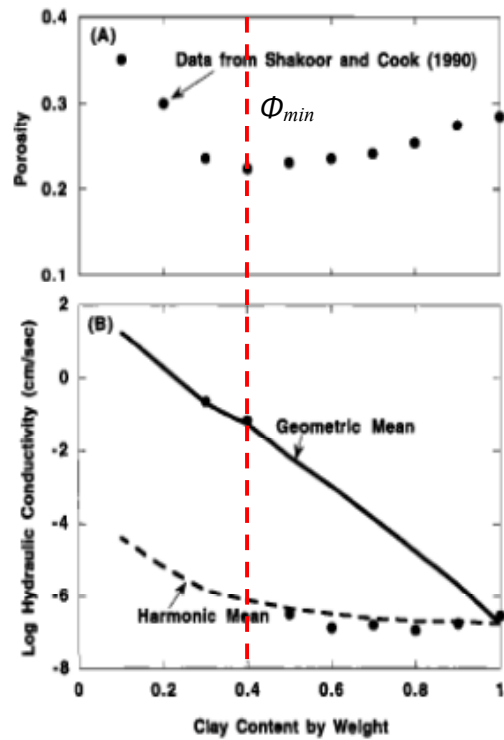
When sediments of two sizes are mixed together,  $\Phi$  varies non-linearly (Figure 1) and a minimum  $\Phi$  will be reached for some proportion of sediments (Koltermann and Gorelick, 1995). Theoretically, the  $\Phi_{min}$  will occur when the volume of the finer-grained component equals the pore volume of the unmixed coarser-grained component.  $\log(k)$  also changes non-linearly at the change from coarse to fine packing. In real sediment mixtures, sediments will pack in a way that differs from ideal packing (Koltermann and Gorelick, 1995).

Figure 2 and Figure 3 (Koltermann and Gorelick, 1995) show how  $\Phi$ ,  $\log(k)$ , and  $\log$  saturated hydraulic conductivity ( $K$ ) vary in sand and clay mixtures. (Note that for a specific fluid density and viscosity the relationships of  $\log(K)$  and  $\log(k)$  to the mixture fraction are the same.) These data provide evidence of what Koltermann and Gorelick (1995) refer to as an “abrupt drop in hydraulic conductivity [that] occurs on either side of the porosity minimum.” Koltermann and Gorelick (1995) explained this abrupt drop by focusing on the geometry and size of the pores that dominate fluid flow. If a mixture contains a low fraction of finer grains then coarse packing exists, so the coarser grains maintain their original pore structure and the “hydraulic conductivity of the coarse-grained component ... exert[s] an influence” (Koltermann

and Gorelick, 1995). Mixtures in which clusters of unoccupied coarser pores are connected exhibit greater hydraulic conductivity. However, if a mixture contains a high fraction of finer grains then the pore structure is dominated by finer grains so fine packing exists, and the hydraulic conductivity is close to that of the finer-grained component (Figures 1, 2, and 3).



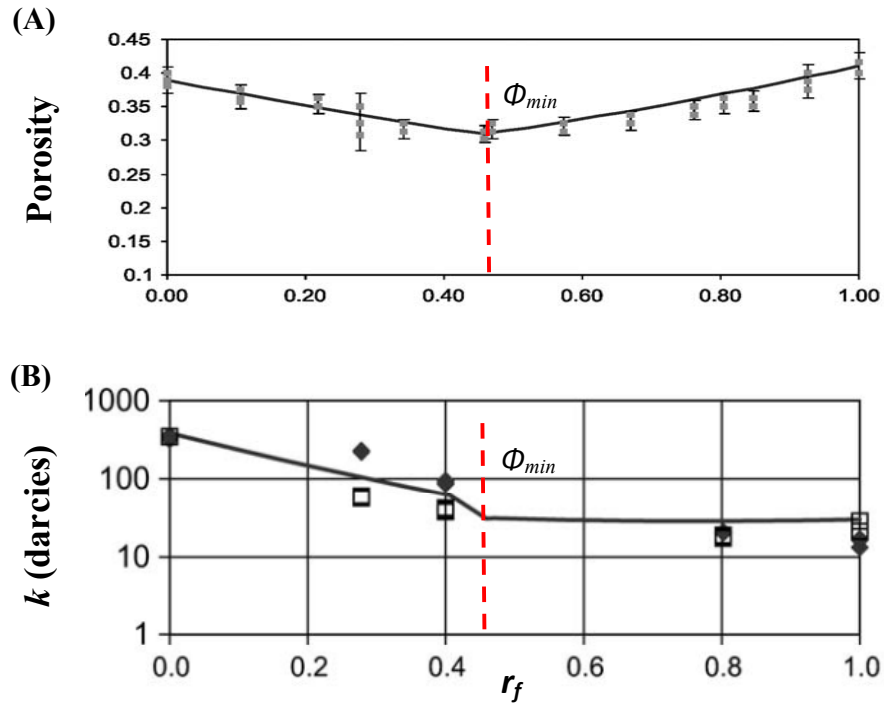
**Figure 2.** (A):  $\Phi$  vs. clay content by weight. (B): Measured  $\log(k)$  (circles) plotted with  $k$  modeled using the harmonic mean (dashed line) and the geometric mean (solid line) for grain diameter in the Kozeny-Carman equation (from Koltermann and Gorelick, 1995).



**Figure 3.** (A):  $\Phi$  vs. clay content by weight. (B): Measured  $\log(K)$  (circles) plotted with  $K$  modeled using the harmonic mean (dashed line) and the geometric mean (solid line) in the Kozeny-Carman equation (from Koltermann and Gorelick, 1995).

Kamann et al. (2007) presented results from experiments similar to those analyzed by Koltermann and Gorelick (1995). In this study fine sand was mixed with coarse sand, among mixtures of other grain sizes. Figure 4 (Kamann et al., 2007) shows how porosity varied as a function of the volume of finer grains per volume of mixed sediment ( $r_f$ ), and how  $\log(k)$  varied. The abscissa in Figures 2 and 3 (weight fraction) are related to the abscissa in Figure 4 (volume fraction of fines,  $r_f$ ) through equation (2) in Koltermann and Gorelick (1995). Both represent the relative abundance of the finer-grained fraction in a mixture.

Note that in Figure 4B, the measured data indicate a change in slope, but with resolution insufficient for determining where the change in slope occurs. Five other experiments by Kamann et al. (2007), using other grain sizes, also had insufficient data around  $\Phi_{min}$  for determining where the slope changes.

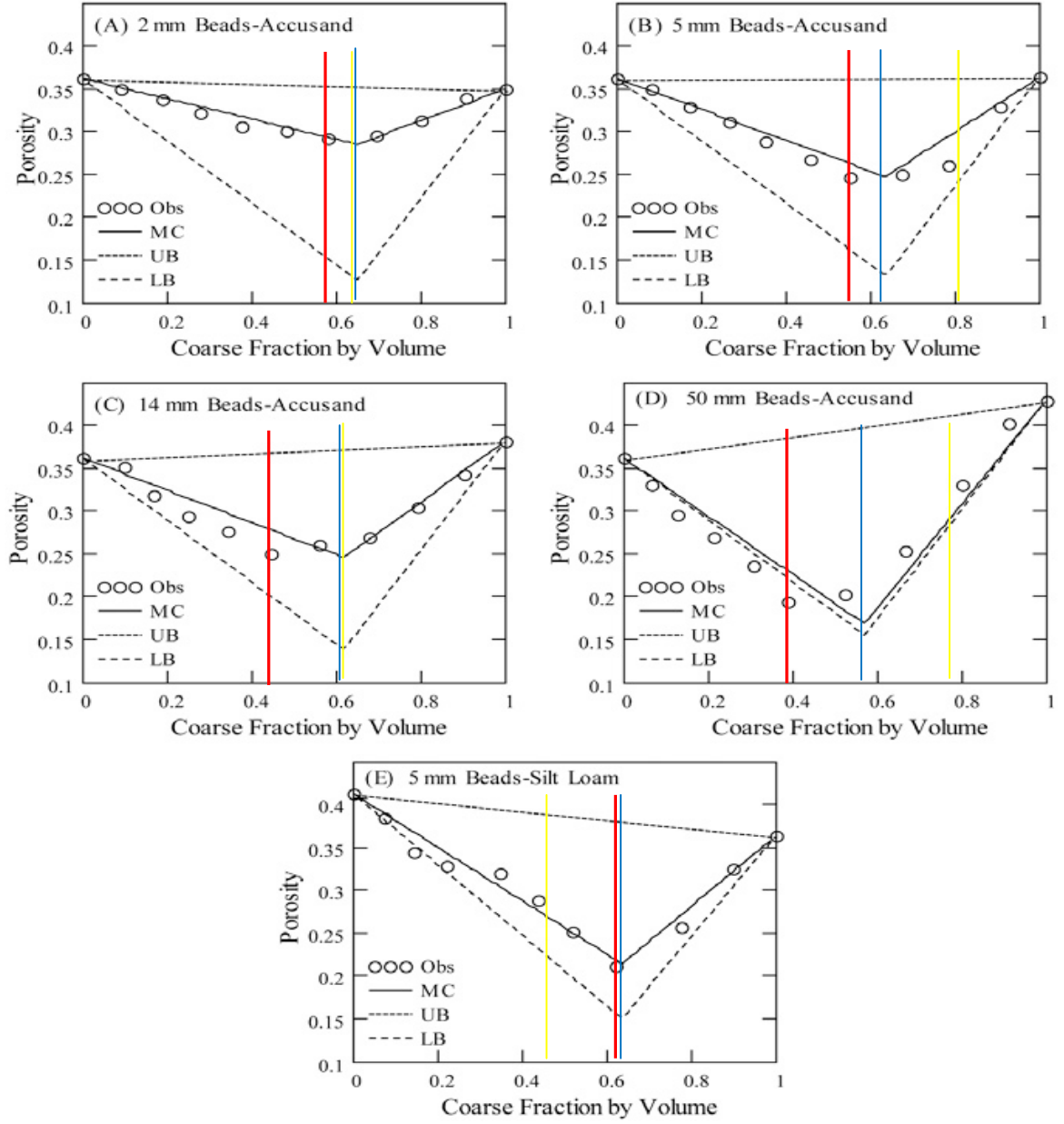


**Figure 4. (A):** Measured and modeled (solid line) porosity for fine and coarse sand mixtures (Kamann et al., 2007).  
**(B):** Measured and modeled (solid line)  $k$  for fine and coarse sand mixtures (from Kamann et al., 2007).

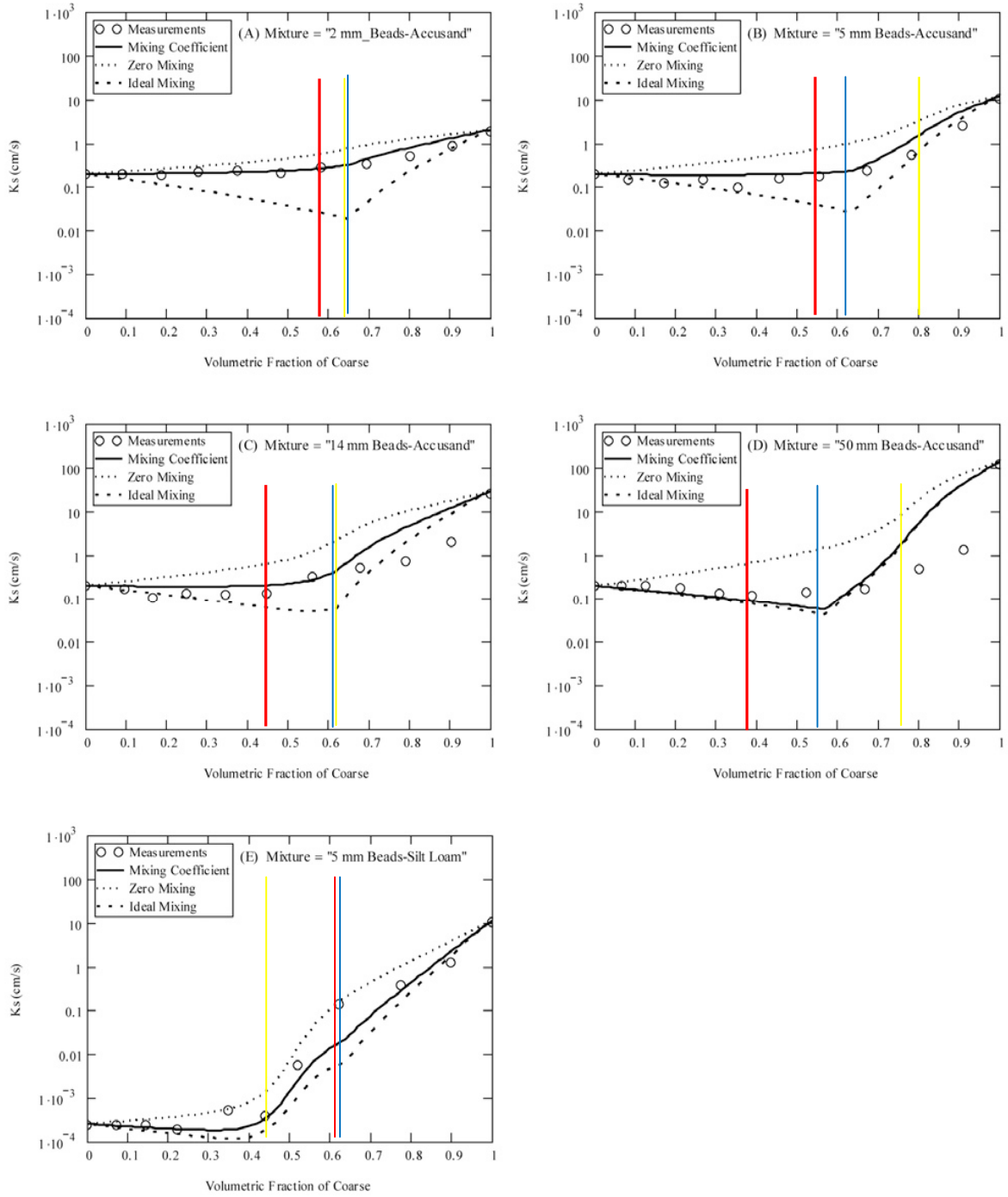
Zhang et al. (2010) showed results from experiments similar to those shown by Koltermann and Gorelick (1995) and Kamann et al. (2007). They used glass beads that represented gravel or sand, and a natural silt loam (approximately 70% silt, 15% clay, and 15% sand). Figure 5 and Figure 6 show how  $\Phi$  and  $\log(K)$  changed, respectively, as a function of the volume fraction of coarser grains. The abscissas in Figures 5 and 6 use a different metric to represent the relative abundance of each grain size in a mixture. The “Coarse Fraction” is the volume of all coarse grains plus the volume of unfilled coarse pores per volume of mixed sediment. Thus, the “Coarse Fraction” is equal to  $1-r_f$ .

As illustrated in Figure 6,  $\log(K)$  for mixtures with low coarse fractions is relatively low and is approximately equal to the  $\log(K)$  of the finer-grained component. However, there is a coarse fraction at which the slope in the relationship of  $\log(K)$  and the mixture fraction changes. Zhang et al. (2010) refer to this point as the “critical mixture fraction” and state that this change in slope occurs because “connected large pores were formed among the gravels.”

Figures 5E and 6E show data that suggest the slope of the relationship changes abruptly and that the abrupt change occurs well below the coarse mixture fraction at which  $\Phi_{min}$  occurs. Figures 5A-5D and 6A-6D show data that suggest the slope of the relationship changes close to the measured coarse fraction at which  $\Phi_{min}$  occurs, but the nature and location of the change are not as distinct as in Figures 5E and 6E. Figures 5C and 5D show measured  $\Phi_{min}$  at a coarse mixture fraction close to 0.4, although Zhang et al. (2010) modeled  $\Phi_{min}$  to be close to 0.6. Zhang et al. (2010) stated that the critical mixture fraction at which the  $\log(K)$  versus mixture fraction slope change occurs is strongly dependent on mixing and packing procedures. Furthermore, they stated that the critical mixture fraction can only be empirically determined, and can vary widely from the mixture fraction at which  $\Phi_{min}$  occurs.



**Figure 5.** Measured (circles) and modeled (lines)  $\Phi$  versus volume fraction of coarse grains for five sediment mixtures. Solid and dashed lines (MC, UB, and LB) represent different modeling approaches. Red lines represent measured  $\Phi_{min}$ , blue lines represent modeled  $\Phi_{min}$ , and yellow lines represent  $b_{vco}$  (Zhang et al., 2010).



**Figure 6.** Measured (circles) and modeled (lines) saturated  $K$  ( $K_s$  on their axis label) versus volume fraction of coarse grains for five sediment mixtures. Red lines represent measured  $\Phi_{min}$ , blue lines represent modeled  $\Phi_{min}$ , and yellow lines represent  $b_{vc0}$  (Zhang et al., 2010).

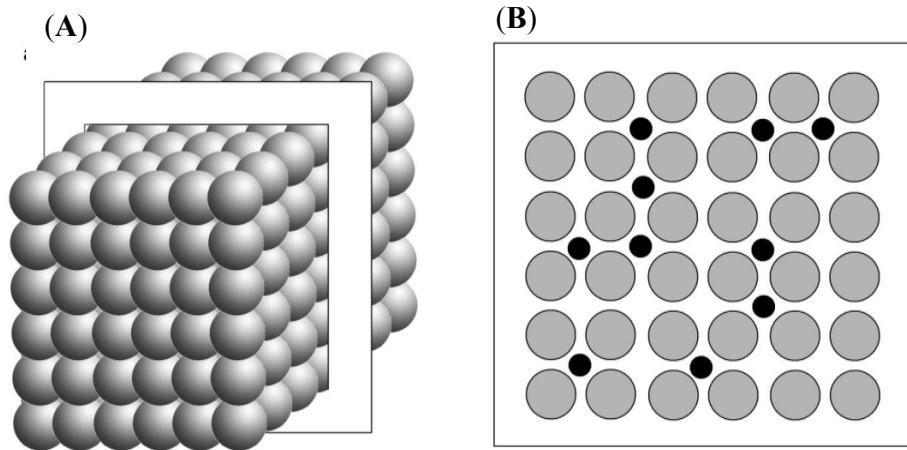
Porter et al. (2012) stated that an abrupt change in hydraulic conductivity should occur at a mixture fraction corresponding to a percolation threshold ( $\omega_c$ ), rather than to  $\Phi_{min}$ . They used ideas from percolation theory to support their argument. Percolation theory (Stauffer and Aharoney, 1994) addresses issues related to the interconnection of unoccupied pores. Figure 7 is a graphical representation of how percolation theory can be used to characterize a network of pores (Porter et al., 2012). Percolation theory holds that a connected network of unoccupied coarser pores will span the sediment volume and percolate so long as the fraction of occupied coarser pores ( $\omega$ ) is below a threshold value ( $\omega_c$ ).

Two percolation models that can be applied to pore networks are site percolation on a simple cubic lattice (Figure 7) and continuum percolation of penetrating spheres (Porter et al., 2012). Considering a situation of perfect cubic packing of spheres, Porter et al. (2012) stated that “the pore network could be thought of as a lattice of pores, each with connection to six pores surrounding it.” For site percolation on a simple cubic lattice,  $\omega_c$  equals 0.69 (Stauffer and Aharoney, 1994). In the case of continuum percolation of penetrating spheres, the model consists of spatially uncorrelated uni-modal spheres (representing pore space), where spheres are allowed to overlap, and  $\omega_c$  equals 0.28 (Lorenz and Ziff, 2001; Consiglio et al., 2003).

The  $\omega$  equals unity when all coarse pores are occupied by finer grains. Under ideal packing, this would occur for the  $r_f$  at  $\Phi_{min}$ . When  $r_f$  is less than that at which  $\Phi_{min}$  occurs there are percolating clusters of unoccupied coarser pores over the range of  $r_f$  at which  $\omega \leq \omega_c$ , and no continuous pathways of unoccupied coarser pores for  $r_f$  at which  $\omega_c \leq \omega \leq 1$ . If an abrupt change in  $\log(k)$  is caused by the disruption in the continuity of unoccupied coarser pore pathways, then it should occur at the  $r_f$  corresponding to  $\omega_c$ , and thus at an  $r_f$  less than that at which  $\Phi_{min}$  occurs.

Real mixtures do not have pore networks that are simple cubic lattices or overlapping spheres as in the percolation models, and thus thresholds equal to 0.69 or 0.28 are not expected.

Porter et al. (2012) explored the idea that some  $\omega_c$  might be identified for real mixtures by studying measured  $\log(k)$  in mixtures that included from three to sixteen different grain sizes. The mixtures used by Porter et al. (2012) were such that the finer grains were similar in size to the coarser pores. The relationship between  $\log(k)$  and grain size was better modeled if a percolation threshold was included, with  $\omega_c$  equal to 0.8. However, the study did not involve a systematic variation of the volume fraction of fines within binary mixtures, and thus the results could not be used to define the relationship illustrated in Figure 1 nor the existence of a specific threshold value for  $r_f$  as related to  $\omega_c$ .



**Figure 7.** (A) Idealized cubic packing of spherical grains. (B) Cross section of pore network. Black circles represent occupied pores. Percolation theory states that there will be a global connection of unoccupied pores across the system if the proportion of occupied pores is less than a certain threshold ( $\omega_c$ ). From Porter et al. (2012).



In summary, three ideas have been presented in the literature about the change in slope in the relationship between  $\log(k)$  and the mixture fraction: (1) change at the  $r_f$  corresponding to  $\Phi_{min}$  (Koltermann and Gorelick, 1995); (2) change at the  $r_f$  corresponding to  $\omega_c$  (Porter et al., 2012); and (3) change at an  $r_f$  that can only empirically be determined (Zhang et al., 2010).

The goal of this study was to explore the change in slope in the relationship between  $\log(k)$  and the mixture fraction of fines in mixtures of two grain sizes. The objective was to collect data with high resolution of the relationship between  $\log(k)$  and the volume fraction of fines near the volume fraction at which  $\Phi_{min}$  occurs and evaluate which of the three ideas from previous studies are supported. The experiments by Kamann et al. (2007), as plotted in Figure 4 were extended to collect more data for  $r_f$  between 0.2 and 0.4.

### 1.1 Kozeny-Carman Model

All three studies discussed above were focused on the use of the Kozeny-Carman (K-C) model in ways corresponding to the relationship between  $\log(k)$  and the mixture fraction. The K-C model gives representative values of  $k$  as a function of  $\Phi$  and an effective grain diameter ( $d_e$ ):

$$k = \frac{d_e^2 \phi^3}{180(1-\phi)^2} \quad (1)$$

(Koltermann and Gorelick, 1995; Esselburn, 2010; Porter et al., 2012). The  $d_e$  has been calculated in different ways. Chapuis (2003) used the fraction-weighted harmonic mean of grain sizes in a mixture for  $d_e$ . Koltermann and Gorelick (1995) used the fraction-weighted geometric mean if  $r_f$  was less than that at which  $\Phi_{min}$  occurs and the harmonic mean if the  $r_f$  was greater

than that at which  $\Phi_{min}$  occurs. Porter et al. (2012) used the geometric mean at  $r_f$  less than the  $r_f$  that corresponds to  $\omega_c$  and the harmonic mean if the  $r_f$  was greater than the  $r_f$  that corresponds to  $\omega_c$ .

Zhang et al. (2010) implied that there should not be a discontinuity in the calculation of  $d_e$  as there is in the Koltermann and Gorelick (1995) and Porter et al. (2012) methods. They used a calculation of  $d_e$  that is continuous across an empirically determined threshold with a power-averaging method.

$$d_e = (b_{vc}d_c^p + b_{vf}d_f^p)^{1/p} \quad (2)$$

where  $b_{vc}$  and  $b_{vf}$  are the post-mixed volume fractions of the coarse and fine components,  $d_c$  and  $d_f$  are the diameters of the coarse and fine grains, and  $p$  is a continuous sigmoidal function:

$$p = \frac{1}{1 + \exp[a(b_{vc0} - b_{vc})]} - 1 \quad (3)$$

where  $a$  is a shape factor, and  $b_{vc0}$  is an empirically determined critical coarse fraction at which  $p=0.5$ .

After exploring the relationship between  $\log(k)$  and the mixture fraction with the new data generated in this study, I also consider the ramifications of those results with regard to these various methods for computing  $d_e$  in the K-C model.

## 2.0 Methodology

The sediments used in these experiments were spherical glass sand-blasting beads, and were the same as used by Kamann et al. (2007) for the results shown in Figures 4 and 5 and also as used by Conrad et al. (2008). They were sorted into fine and coarse sand sizes, with median grain sizes of 0.163 mm and 0.650 mm. Under the assumptions of theoretical cubic packing, about 49 finer grains will fit into one coarser pore. These sizes were chosen when considering the tradeoff between having a large difference in grain size, which results in significant differences in the  $\log(k)$  of the mixture components (desirable), and having coarse pores so much larger than finer grains that mixtures at low  $r_f$  would be unstable (undesirable).

The  $\Phi$  were measured by adding water to the sediment mixtures and recording the volume of water needed to reach saturation. The volume of sediment mixture for each measurement was 200 milliliters. The prepared mixtures were thoroughly stirred as per Mاسترا (2010), and poured into the vessel used for measuring  $\Phi$ .

A constant-head permeameter was used with de-aired water to measure permeability. A detailed explanation of the method is provided by Mاسترا (2010). A total of 27 permeability measurements were made for each mixture. For each mixture, the hydraulic gradient was changed three times, and three measurements were made for each gradient. The sediments were then dried, re-sorted, and re-mixed to create another “packing” for the same mixture. Permeability was measured for three packings (nine measurements per packing).

To evaluate Equation (1) for  $k$  over all mixture fractions,  $\Phi$  must be represented over all mixture fractions. The  $\Phi$  values used in Equation (1) were interpolated between measured values using the approach by Kamann et al. (2007). This representation is a piecewise-linear function:

$$\begin{aligned}
\Phi &= \Phi_c - \left[ \frac{\Phi_c - \Phi_{min}}{\xi_{\Phi_{min}}} \right] \xi; \quad \xi \leq \xi_{\Phi_{min}} \\
\Phi &= \Phi_f - \left[ \frac{\Phi_f - \Phi_{min}}{1 - \xi_{\Phi_{min}}} \right] (\xi - 1); \quad \xi \geq \xi_{\Phi_{min}}
\end{aligned}
\tag{4}$$

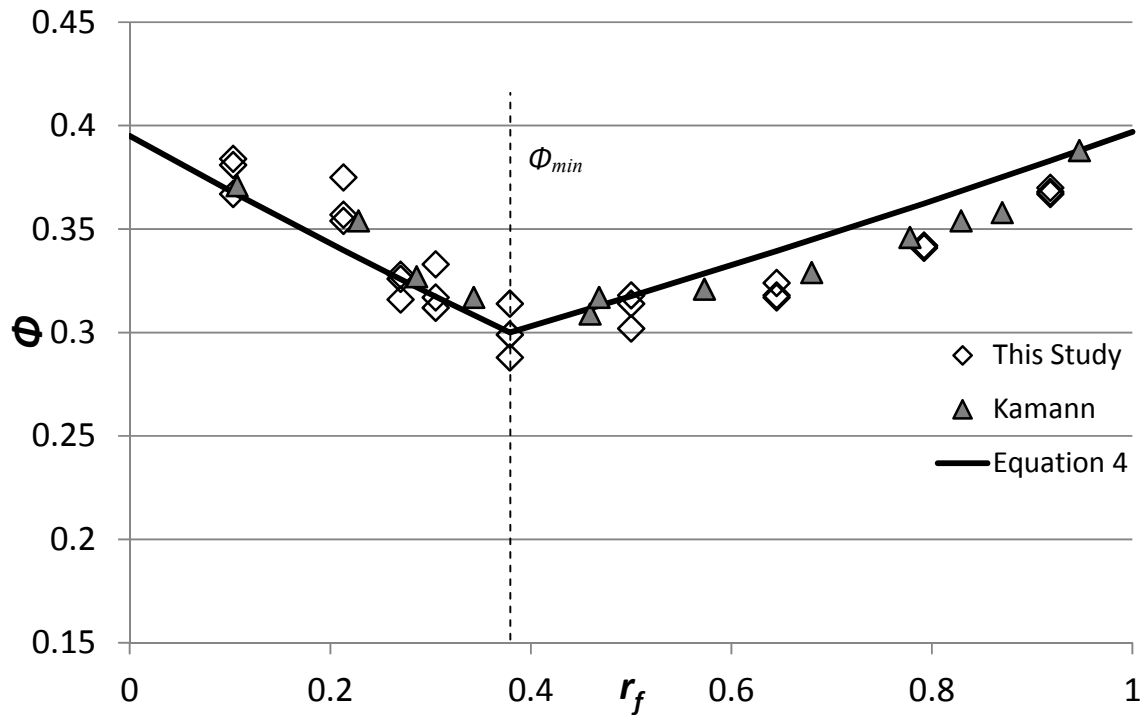
where  $\xi$  is the pre-mixed volume fraction of fines,  $\Phi_c$  and  $\Phi_f$  are the experimentally measured porosities of the coarse and fine-grained sand, and  $\xi_{\Phi_{min}}$  is the premixed volume fraction of fines that results in  $\Phi_{min}$ . Equation 4 requires that  $\Phi_c$  and  $\Phi_f$  and  $\Phi_{min}$  be known. Other piecewise linear methods requiring these values, and in some cases including more variables, have been used in the literature (Koltermann and Gorelick, 1995; Zhang et al., 2010). However, the differences are not significant in this context and so these other methods of representing  $\Phi$  are not used or discussed further here.

### 3.0 Results and Discussion

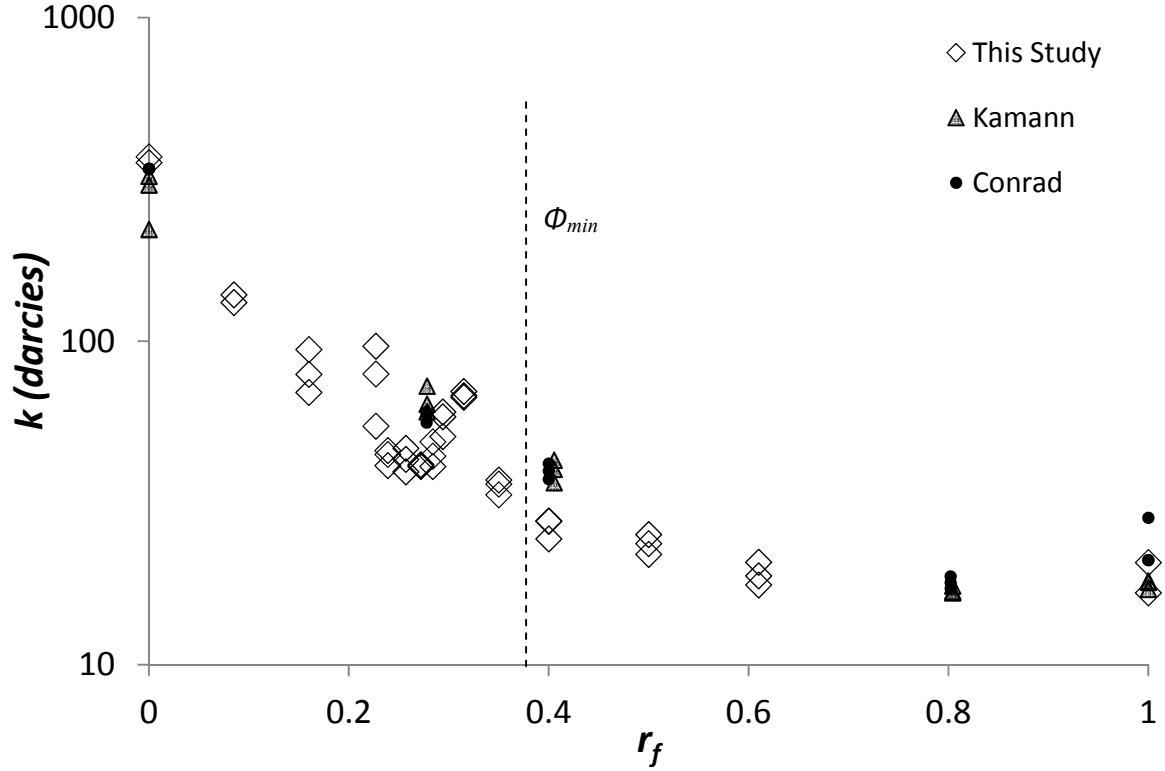
Figure 8 shows measured  $\Phi$  from this study as well as from Kamann (2004). The  $\Phi_{min}$  occurs when  $r_f = 0.38$ , which is approximately equal to  $\Phi_c$ . This is the  $r_f$  value that will be referred to in further discussion about where  $\Phi_{min}$  occurs. Figure 8 also shows Equation (4).

Figure 9 shows measured  $k$  from this study together with data from Kamann (2004) and Conrad (2006). As the proportion of finer grains is increased from zero,  $\log(k)$  decreases linearly until  $r_f = 0.4$ . There is no change in slope indicated when  $r_f$  is below that at which  $\Phi_{min}$  occurs (0.38). Therefore, the data show no evidence of an abrupt change in slope at the  $r_f$  that corresponds to a percolation threshold. In fact, Figure 9 shows evidence for a change in slope when  $r_f$  is slightly greater than 0.38. Figure 6E showed a clear change in slope at an  $r_f$  above the  $r_f$  at which  $\Phi_{min}$  was measured.

Zhang et al. (2010) attributed the idea that the critical mixture fraction is not consistent in relation to  $\Phi_{min}$  for various sediment mixtures to variations of mixing and packing, and inferred that the critical mixture fraction must be empirically determined for each mixture. This idea is supported by the variability among the results seen in Figures 2, 3, 6A-E, and 9.



**Figure 8.** Porosity ( $\Phi$ ) results from the expanded fractional-packing model (Equation 4) plotted with measured  $\Phi$  data from this study and Kamann (2004).



**Figure 9.** Measured  $k$  from this study and Kamann et al. (2007) and Conrad et al. (2006).

A percolation threshold ( $\omega_c$ ) was not applicable to measured data from this study, yet was applicable to mixtures analyzed by Porter et al. (2012). The discordance between results of these studies can be explained by considering the way finer grains occupied coarser pores. When sediment grains of two sizes are mixed together, a coarser pore can either be fully occupied by finer grains, partially occupied by finer grains, or unoccupied. Furthermore, finer grains can be distributed uniformly or non-uniformly throughout all coarser pores. In the case of uniform distribution, all coarser pores are occupied with the same amount of finer grains. In the case of non-uniform distribution, the amount of finer grains that occupy each coarser pore is variable.

Porter et al. (2012) introduced a variable,  $m^{j,i}$ , that describes how many finer grains (category  $i$ ) can fit into one coarser pore (category  $j$ ):

$$m^{j,i} = \frac{V_{p_j}}{V_{g_i}} \quad (5)$$

where,  $V_{p_j}$  is the volume of one coarser pore, and  $V_{g_i}$  is the volume of one finer grain. When  $m^{j,i}$  equals 2.32, exactly one spherical finer grain can fit into a coarser pore under cubic packing of spherical grains (Porter et al., 2012).

The value of  $m^{j,i}$  is important when considering whether or not ideas of percolation theory can be logically applied or not to a particular mixture. If  $m^{j,i}$  is close to 2.32, then each coarser pore is either occupied by one finer grain or unoccupied, in a binary fashion. This is the situation considered in site percolation theory.

If  $m^{j,i}$  is greater than 2.32, then more than one finer grain is able to fit into each coarser pore. In this case, each coarser pore can be fully occupied, partially occupied, or unoccupied by finer grains. The condition of partially occupation of coarser pores is not considered in percolation theory, thus ideas from percolation theory would not be logically applied in this case.

A percolation threshold was likely important for explaining the results of Porter et al. (2012) because the  $m^{j,i}$  values of those mixtures were close to 2.32. For data reviewed in this study (Figures 2, 3, 6A, 6B, 6C, 6D, 6E, and 9) the values of  $m^{j,i}$  for the mixtures were significantly greater than 2.32 (Table 1). Therefore, the ideas from percolation theory would not be logically applied to these mixtures.

**Table 1.**  $m^{j,i}$  for sediment mixtures in Figures 2, 3, 6, and 9.

	$d_g^j$ (mm)	$d_g^i$ (mm)	$V_g^i$ (mm <sup>3</sup> )	$V_p^j$ (mm <sup>3</sup> )	$m^{j,i}$
<b>Figure 2*</b>	0.5	0.0006	1.13E-10	0.040545	3.58E+08
<b>Figure 3*</b>	16	0.001	5.24E-10	1868.304	3.57E+12
<b>Figure 6A</b>	2	0.713	0.189787	2.830026	14.9
<b>Figure 6B</b>	5	0.713	0.189787	45.99299	242
<b>Figure 6C</b>	14	0.713	0.189787	1059.703	5580
<b>Figure 6D</b>	50	0.713	0.189787	54355.36	286000
<b>Figure 6E</b>	5	0.039	3.11E-05	45.99299	1480000
<b>Figure 9</b>	0.65	0.163	0.002268	0.111346	49.1

\*Values approximated based on Koltermann and Gorelick (1995).

### 3.1 Implications for the Kozeny-Carman Model

Figure 10 shows measured  $k$  from this study, from Kamann et al. (2007) and from Conrad et al. (2008). The K-C model is also shown as computed with  $d_e$  equal to the harmonic mean grain size,  $d_e$  equal to the geometric mean grain size, and  $d_e$  calculated with Equation 2. In calculating Equation 2, the values of 20 and  $(1-0.5\Phi_c)$  for empirical parameters  $a$  and  $b_{vco}$  gave



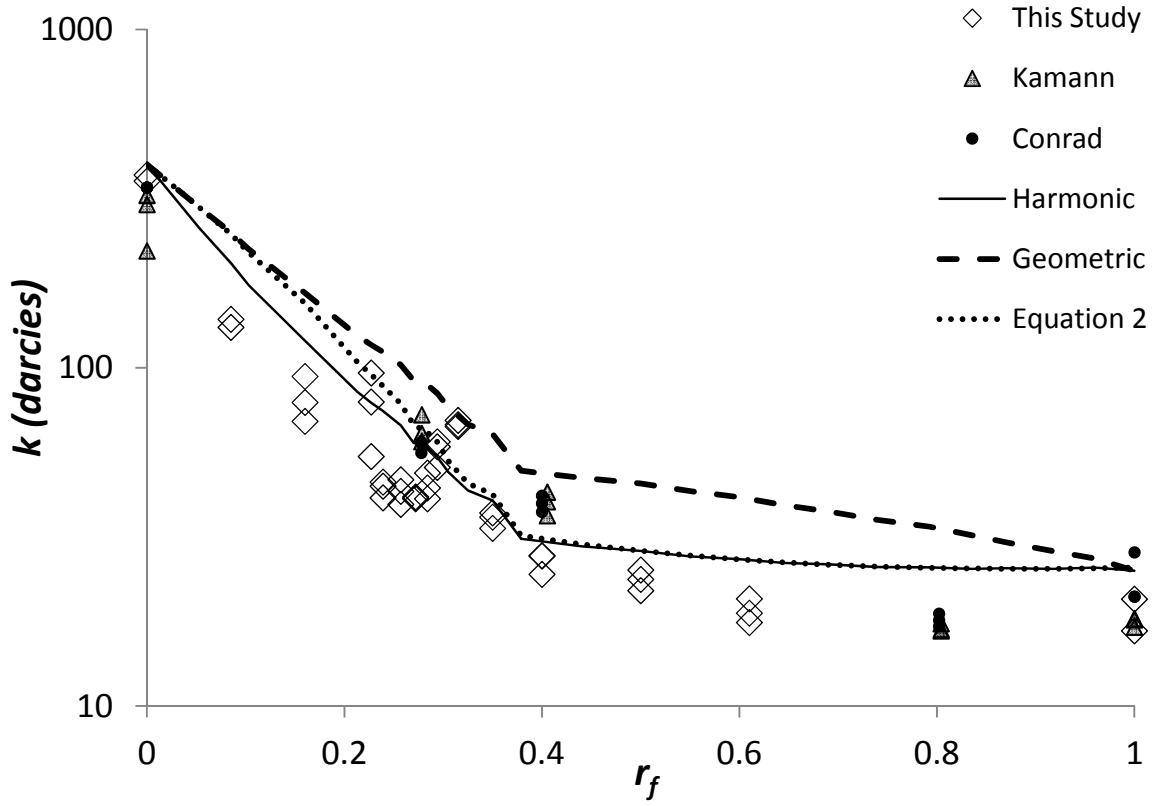
the best results and are used for the figure. The model computed with each method for  $d_e$  closely represents the measured data.

The root mean square error (RMSE) was computed for each of the three methods for computing  $d_e$ . The error is taken as the difference between the logarithm of the computed and observed permeability. The results are given in Table 2. According to Figure 10 and RMSE comparisons, using the harmonic mean to compute  $d_e$  over all  $r_f$ , as per Chapuis (2003), works best for this mixture type (fine and coarse sand).

**Table 2.** RMSE values for three methods for calculating  $d_e$ .

Method for $d_e$	RMSE
Harmonic	0.1405
Geometric	0.2802
Equation 2	0.1819

All three of the methods over-predicted  $k$  at low and high values of  $r_f$ . This may be due to packing procedures and higher degrees of non-ideal mixing in these regions, where it is harder to make uniform mixtures and thus unmixed clusters of fine and coarse sediment are more likely (Zhang et al, 2010). It may also be due to the limitations of the K-C equation itself.



**Figure 10.** Measured  $k$  data compared with the K-C model. The model was computed with three different methods for computing  $d_e$ .

#### 4.0 Conclusions

In sediment mixtures of two grain-size components, porosity and permeability vary non-linearly as a function of the grain size and the volume fraction of each component. These relationships have been described in the literature using a conceptual model that is based on ideal (uniform) packing which has, in turn, been used as a basis for semi-empirical, quantitative models for those relationships. Under ideal packing, the porosity minimum ( $\Phi_{min}$ ) occurs at the mixture fraction for which the volume of the finer grains equals the original pore volume of the coarser grains.

Real mixtures have non-ideal packing and the observed  $\Phi_{min}$  is usually not as low as the ideal  $\Phi_{min}$ , though they commonly occur at the same  $r_f$ . In real mixtures the slope of the  $\log(k)$  versus  $r_f$  relationship commonly has an abrupt change. The abrupt change in slope that has been observed in the  $\log(k)$  versus  $r_f$  relationship should occur at the  $r_f$  where coarser pore pathways change from continuous to discontinuous. For mixtures in which finer grains are approximately the size of coarser pores, unoccupied coarser pores will percolate (connect across the sample) if the fraction of occupied pores ( $\omega$ ) is below a percolation threshold ( $\omega_c$ ), and not percolate if  $\omega$  is greater than  $\omega_c$ . An abrupt change in slope of the  $\log(k)$  versus  $r_f$  relationship should occur at the  $r_f$  at which  $\omega$  equals  $\omega_c$ , which is less than the  $r_f$  at which the  $\Phi_{min}$  occurs.

Fine sand was mixed with coarse sand at different volume fractions, and the abrupt slope change was observed to occur at an  $r_f$  above the  $r_f$  at which  $\Phi_{min}$  occurred (Figure 9). Among similar experiments in the literature, the change in slope was observed to occur at  $r_f$  less than, equal to, or greater than the  $r_f$  at which  $\Phi_{min}$  occurs, but commonly occurs at  $r_f$  relatively close to that of  $\Phi_{min}$ .

These differences can be explained by considering the relative sizes of finer grains and coarser grains in a mixture, which govern how the finer grains are distributed within the coarser pores. For mixtures in which finer grains are approximately the same size as coarser pores, coarser pores are either fully occupied or unoccupied by finer grains in a binary fashion. In such mixtures, concepts from percolation theory should be valid, and an abrupt slope change in the  $\log(k)$  – volume fraction relationship would be expected to occur at a percolation threshold. For mixtures in which finer grains are much smaller than coarser pores, concepts from percolation theory would not be valid. The differences in where the slope of the  $\log(k)$  – mixture fraction

relationship changes among different mixtures is likely due to non-ideal mixing and variations in packing procedures.

The literature contains ideas for using the Kozeny-Carman equation to model the  $\log(k)$  versus  $r_f$  relationship. Of the methods reviewed from the literature, they all represented  $k$  as it varies per volume fraction of finer grains relatively well.

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## APPENDIX A – Methods for Sediment Preparation

### Sorting Procedures

Sediment was sorted into fine and coarse sand sizes with a set of nested sieves. Based on the sieves used, the range of grain diameters for fine sand was 0.149 to 0.177 mm and for coarse sand was 0.590 to 0.710 mm. The nested sieves were loaded into a Ro-tap® sieve shaker for 15 minutes (Figure A1). Caution was taken to avoid any effect from static electricity to occur. To prevent the sediment from sticking together due to static electricity, rubber stoppers and larger glass marbles were placed in each sieve to keep the beads separated. Also, the Ro-tap® was grounded while running to limit any static charge. After measurements were completed, the sediment was oven-dried, which lead to the formation of aggregates of beads. These were effectively broken up during the sieving process. Any aggregates that remained on the top sieve were discarded.



**Figure A1.** Ro-tap® sieve shaker used for sorting sediment.

### **Sediment Preparation for Permeability Measurements**

Sediment was measured and poured into a large container to be mixed. While pouring sediment into the container, the mixture was stirred with a wooden spoon periodically. The container with the complete mixture was then shaken manually for about 5 minutes. The sediment was then poured into the permeameter chamber with a 250 milliliter beaker. Sediment was poured in a circular motion around the edges of the chamber.



## APPENDIX B – Permeability Measurements

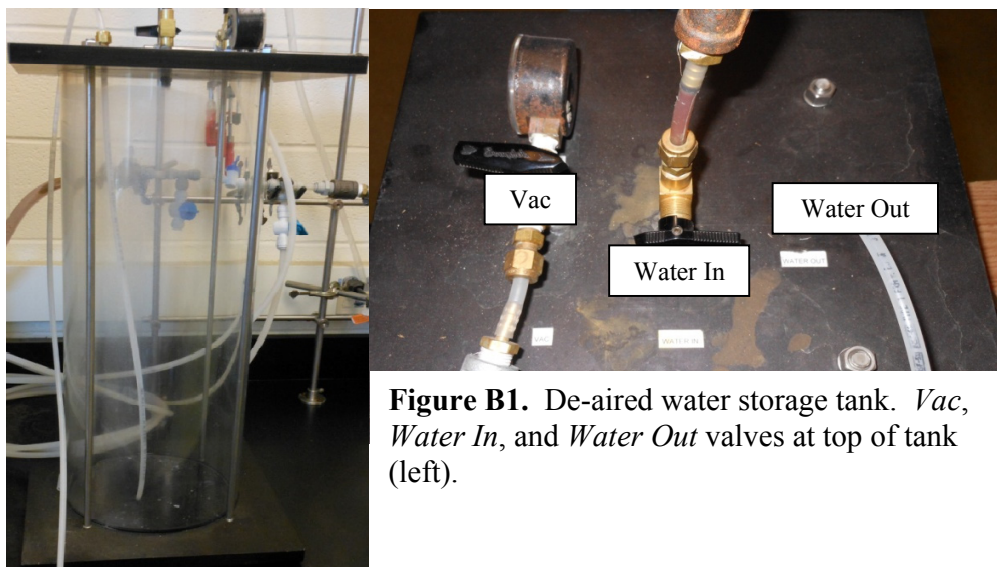
### Water De-airing Procedure

A vacuum pump was used to de-air water for permeability measurements. The de-aired water storage tank is shown in Figure B1. The following steps were taken in to de-air the water:

- (1) Turn vacuum pump on and open series of valves that connect to the *Vac* valve.
- (2) Let vacuum pump run for several minutes.
- (3) Turn water on and open *Water In* valve. Allow water storage tank to fill.
- (4) Turn off water to storage tank.
- (5) Let vacuum run for additional 8 to 12 hours.

*To Empty storage tank:*

- (6) Open *Vac* valve, then open series of valves so the connecting tube is open to air.
- (7) Open *Water Out* valve and empty water into the tank to be used with the permeameter set-up.



**Figure B1.** De-aired water storage tank. *Vac*, *Water In*, and *Water Out* valves at top of tank (left).

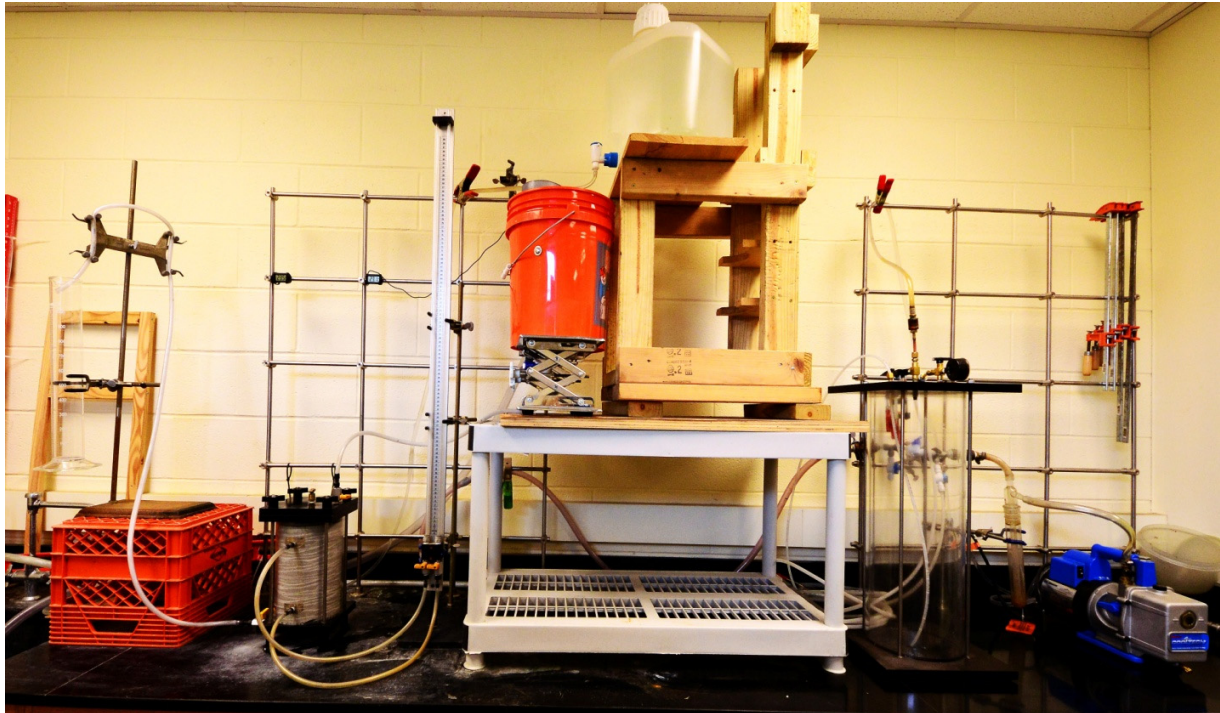
### Permeameter Procedure

A detailed, step-by-step procedure for measuring permeability is given by Mastera (2010). The constant head permeameter (Figure B2) directly measures the hydraulic gradient ( $\Delta h/\Delta l$ ), volumetric flow rate ( $Q$ ), and temperature ( $^{\circ}\text{C}$ ) of the water. Temperature was measured to find the fluid density ( $\rho$ ) and fluid viscosity ( $\mu$ ) of the water. These variables were then used in the following equation to determine permeability ( $k$ ):

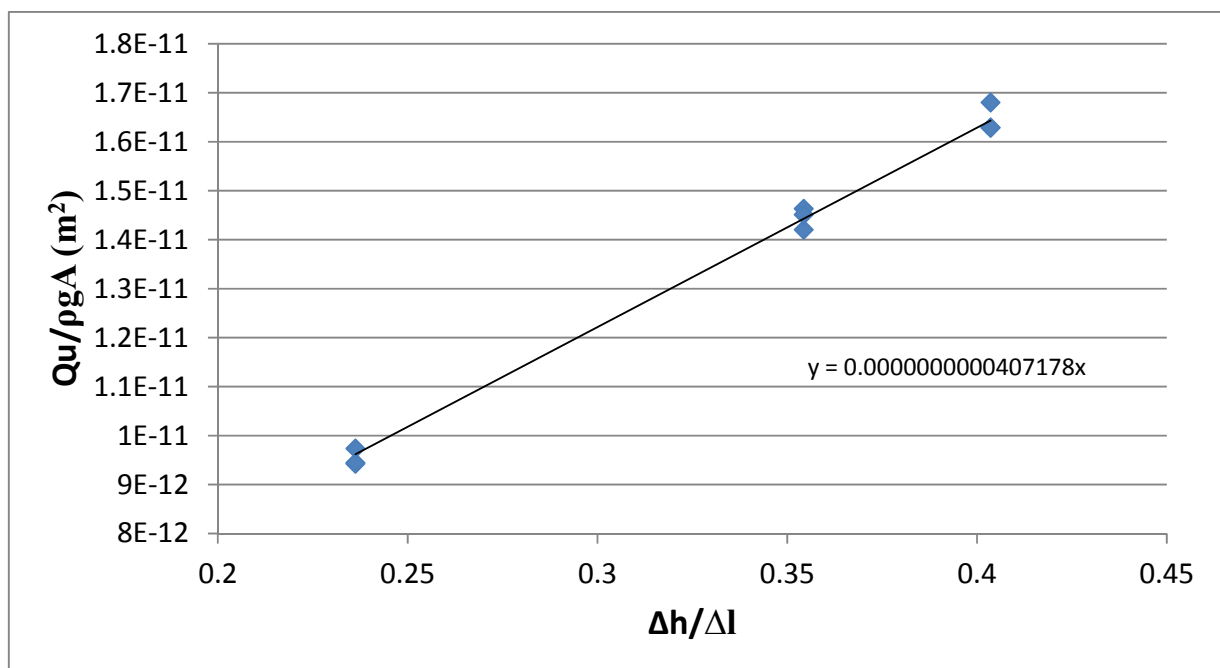
$$k = Q \left( \frac{\rho g A}{\mu} \frac{\partial h}{\partial l} \right)^{-1} \quad (\text{B1})$$

where  $A$  is the cross sectional area of the sediment mixture and  $g$  is gravitational acceleration.

To ensure the validity of Darcy's law during the measurements and subsequent calculations,  $Q\mu/\rho g A$  vs.  $(\Delta h/\Delta l)$  was plotted. The relationship between  $Q\mu/\rho g A$  and  $(\Delta h/\Delta l)$  must be linear for Darcy's law to be valid. If the relationship was not linear, permeability for that mixture was re-measured. The slope of this line, if linear, was the average  $k$  value for that packing. An example of this plot is shown in Figure B3.



**Figure B2.** Apparatus used for measuring permeability.



**Figure B3.** Sample plot of  $Qu/\rho g A$  versus the hydraulic gradient ( $\Delta h/\Delta l$ ) for measured  $k$ .